AN ANALYSIS OF FACTORS AFFECTING DYNAMIC RESPONSE OF GEAR TRANSMISSIONS OF SERVOMECHANISMS

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Abstract: The paper presents the main aspects of dynamic modeling of geared systems in relation to the influence factors of nonlinearities of the servomechanisms. The nonlinearities caused of these influence factors can reduce the system stability and also decrease the repeatability and accuracy of geared servomechanisms. The effects of mesh stiffness, backlash and tooth friction are considered in these models.

Keywords: Servomechanism, spur gears, mesh stiffness, gear backlash.

1. Introduction

Gear transmissions are used in the structure of servomechanisms to achieve high driving torques. But these gear systems introduce backlash and friction and reduce the motion accuracy. The dynamic characteristics of gear pairs are significant for the design and control of these devices [1-4], [8], [17], [20]. A literature reveals different analytical models of geared systems according to the control aspects of servomechanisms [3], [5-7],[10-12], [14-16], [19], [23].

The paper presents the main aspects of dynamic modeling of geared systems in relation to the influence factors of nonlinearities of the servomechanisms.

2. Analytical Models of Geared Systems of Servomechanisms

A servomechanism is composed from the servomotor and a gear transmission with the gear ratio \( i \) and is loaded by an effector mechanism (Figure 1).

The literature review attempts to classify dynamic models of gear pairs into groupings with particular relevance in the research presented.

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2.1. Rigid Body Models

According to a rigid body assumption [1], [3], the motor torque $T_m(t)$ can be expressed as

$$ \left( \frac{J_m}{l} + J_G l \right) \ddot{\theta}_G = T_m(t), \quad (1) $$

where $J_m$ is the inertia of the actuator and the first part of transmission, $J_G$ is the inertia of the second part of transmission, $\dot{\theta}_G$ represents the acceleration of the effector and $i$ represents the transmission ratio. This model includes only the effect of inertia components but cannot consider the dynamic interaction between mechanical system and feedback controller due to elastic deformations of meshing teeth or backlash.

2.2. Dynamic Models with Mesh Stiffness Consideration

The varying mesh stiffness has significant influence on dynamic characteristics of gear pairs through the mesh cycle. The mechanical model for a gear pair in mesh is shown in Figure 2. In this model, the teeth are considered as springs and the gear blanks as inertia masses [5], [8], [13]. The differential equations of motion can be expressed as

$$ J_1 \ddot{\theta}_1 + F_d r_{b1} = T_1; \quad (2) $$

$$ J_2 \ddot{\theta}_2 + F_d r_{b2} = T_2, \quad (3) $$

where $\theta_1$, $\theta_2$ are the rotation angle of the pinion and the driven gear, respectively. $J_1$ and $J_2$ are the mass moments of inertia of the gears. $T_1$ and $T_2$ denote the external torques applied on the gear system and $r_{b1}, r_{b2}$ are the base circle radii of the gears. The dynamic load is expressed as

$$ F_d = \sum_{i=1}^{N} F_{di}(t), \quad (4) $$

where

$$ F_{di} = k_i(t) [r_{b1} \dot{\theta}_1 - r_{b2} \dot{\theta}_2] + c(r_{b1} \theta_1 - r_{b2} \theta_2), \quad (5) $$

where $k_i$ represents the mesh stiffness of the gear pair and $c$ is the damping coefficient. By introducing the composite coordinate

$$ x_d = r_{b1} \theta_1 - r_{b2} \theta_2. \quad (6) $$

Eqs. (1) and (2) yield an equation of motion in the following form

$$ m \ddot{x}_d + c \dot{x}_d + \sum_{i=1}^{N} F_{di}(t) = F_n, \quad (7) $$

where $F_n$ is the static load and $N$ represents the number of simultaneous tooth pairs in mesh, and

$$ F_{di}(t) = k_i(t) \left[ x_d + e_i(t) \right]; \quad (8) $$

$$ F_{di}(t) = 0, \quad \text{if} \quad F_{di}(t) < 0, $$

Fig.2. Dynamic model of gear pair
where $e_i(t)$ represents the equivalent error of teeth profile.

The damping coefficient calculated by

$$c = 2\varepsilon \sqrt{m_e k_m},$$

where $m_e$ represents the equivalent inertia mass and $k_m$ is the average mesh stiffness of the gear pair.

These models take into consideration the elastic deformations of engagement teeth, the non-elasting resistance of damping and tooth profile errors.

The large fluctuations of the mesh stiffness are subject of the existence of regimes of dynamic instabilities [11]. An accurate estimation of the gear mesh stiffness is necessary in gear dynamic analysis [10], [12], [15], [18].

### 2.3. Dynamic Models with Backlash Consideration

Backlash between meshing gear teeth can cause impact, reduce system instability, and undesired vibration [1], [4], [8]. By introducing the composite coordinate

$$x_d = n_1 \theta_1 - n_2 \theta_2 - b,$$

where $b$ is the backlash and gear $l$ is the driving gear, the following conditions can occur:

$$x_{d1} > 0.$$  \hspace{1cm} (11.1)

This is the normal operating case and the dynamic tooth load

$$x_{di} \leq 0 \text{ and } |x_{di}| \leq b.$$  \hspace{1cm} (11.2)

In this case the gear will separate and the contact between gears will be lost. Hence, $F_{di} = 0$

$$x_{di} < 0 \text{ and } |x_{di}| > b.$$  \hspace{1cm} (11.3)

In the design stage, the dynamic analysis permits to determine the maximum admisible backlash to satisfy the bandwith requirements of the system with the motor incorporated [1], [2], [4].

### 2.4. Dynamic Models with Backlash and Friction Consideration

In the high precision position control the friction force should be taken into consideration at the formulation of the control law [2], [4], [13]. The basic structure and notations along the line of action are shown in Figures 3 and 4 [8].

![Fig. 3. Double tooth contact](image)

![Fig. 4. Single tooth contact](image)
When contact point occurs in zone AB or DE (Fig. 3), the equations of motion can be written as

\[
J_1 \ddot{\theta}_1 = M_{t1} - F_n r_{b1} - 0.5 \mu F_n p_b - c_1 \dot{\theta}_1 ; \\
J_2 \ddot{\theta}_2 = M_{t2} + F_n r_{b2} - 0.5 \mu F_n p_b - c_2 \dot{\theta}_2 .
\]

(12)

If the contact point occurs in zone BC (Fig. 4), the equations of motion is given by

\[
J_1 \ddot{\theta}_1 = M_{t1} - F_n r_{b1} + \mu F_n l_1 - c_1 \dot{\theta}_1 ; \\
J_2 \ddot{\theta}_2 = M_{t2} + F_n r_{b2} - \mu F_n (l - l_1) - c_2 \dot{\theta}_2 .
\]

(13)

For the case of single-tooth contact in zone CD, the equations of motion are the following

\[
J_1 \ddot{\theta}_1 = T_1 - F_n r_{b1} - \mu F_n l_1 - c_1 \dot{\theta}_1 \\
J_2 \ddot{\theta}_2 = T_2 - F_n r_{b2} - \mu F_n (l - l_1) - c_2 \dot{\theta}_2
\]

(14)

These models are based on the assumption that the load is shared equally among tooth pairs in contact. These assumptions yield the simplified expressions and analytically solutions [4], [16], [20]. Recent formulations include realistic tooth stiffness and the sliding friction over a range of operational conditions [10], [18], [19], [22].

### 3. Dynamic Simulation

Specifications of the pertinent geometrical parameters of the analyzed gear pairs are shown in Table 1, where \( z_1, z_2 \) represent the tooth number of a gear pair, \( x_1, x_2 \) are the addendum modification coefficients and \( m \) is the tooth module.

<table>
<thead>
<tr>
<th>Gear Pair</th>
<th>( z_1 )</th>
<th>( z_2 )</th>
<th>( m ) [mm]</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GP1</td>
<td>18</td>
<td>76</td>
<td>1.5</td>
<td>0.8</td>
<td>-0.82</td>
</tr>
</tbody>
</table>

These parameters are for spur gear pairs having: face-width of gears, \( b = 12 \) [mm] and center distance, \( a = 70 \) [mm]. Additionally, the damping ratio \( \xi = 0.12 \) is considered in the dynamic analysis.

#### 3.1. Mesh Stiffness

For a pair of contacting teeth i, the time-varying mesh stiffness \( k_i(t) \) acts as a parameter excitation.

The teeth pairs in contact act like parallel springs. Therefore, the total mesh stiffness during each engagement cycle can be written as a function of the position of contact point on the action line

\[ k_t = k_s^I + k_s^II, \]

for double-tooth contact

\[ k_t = k_s^I, \]

for simple-tooth contact

where I and II are the mating points of the teeth pairs.

The variation of individual and total mesh stiffness during a mesh cycle is illustrated in Figure 5.

![Variation of mesh stiffness components](image)

**Fig. 5. Variation of mesh stiffness components**

#### 3.2. Analysis of Dynamic Transmission Error

In the analysis of dynamic loads, the transmitting load is defined as \( F_n / b \), where \( F_n \) represents the external static load transmitted by the gear teeth that is proportional to the applied torque \( T_1 \).
A computer program was developed for simulating the dynamic characteristics of spur gear pairs. The equations of motion are solved by the fourth-order Runge-Kutta method. Computer analysis of dynamic characteristics includes different gear pairs with combination of addendum modifications and gear ratio.

The dynamic transmission error $x_d$ is considered in the analysis. Figure 6 shows the variation of dynamic response of spur gear systems with different operation conditions. The effects of backlash and variable mesh stiffness on the amplitude and the variation of dynamic displacement can be analyzed from these data.

The successive impacts correspond to the initial period of operating time in relation to the initial velocity of the pinion.

4. Conclusions

The analytical models presented in the paper show considerable variations in the effects included. These models can include the effect of mesh stiffness, backlash or tooth friction according to the precision control demanded. The nonlinearities caused of these influence factors can reduce the system stability and also decrease the repeatability and accuracy of geared servomechanisms.

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References


