ROLLING FRICTION TORQUE IN MICROSYSTEMS

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ABSTRACT

To determine the rolling friction torque in the micro rolling systems the authors developed an analytical model based on the dissipation of the inertial energy of a rotating microdisc in three rolling microballs. Using an original microrotribomter with two steel rotating discs and three steel micro balls was determined the rolling friction torque in dry conditions for contacts loaded with normal forces of 8.68 mN to 33.2 mN and with rotational speed between 30 to 210 rpm. The experimental results confirm the hypothesis that the rolling friction torque in dry contacts is not depending of the rotational speed.

Keywords: Rolling friction torque, Microtribometer, Dynamic modelling.

INTRODUCTION

The use of the rotating microball bearings in the MEMS applications (micromotors, microgenerators, microactuators, micropumps) implies the simplification in construction, low level of the friction, low level of the wear, high stability, and thus the microball bearings seem to be a promising solution for future MEMS applications.

In the last period some experimental evaluations of the global friction in the rotating microball bearings was realized. Ghalichechian et al. [1] experimentally determined the global friction torque in an encapsulated rotary microball bearing mechanism using silicon micro fabrication and stainless steel microballs of 0.285 mm diameter. The global friction torque was indirectly obtained by measuring the transient response of the rotor in the deceleration process from a constant angular velocity until it completely stops due to friction. Using a high-speed camera system, the angular position of the rotor in the deceleration process was determined. The authors introduced the hypothesis that the global friction torque in the microball bearing is proportional with rotational speed. In this circumstances, the measured angular positions \( \phi(t) \) was fitted to an exponential function in the form \( \phi(t) = a \cdot e^{bt} + c \cdot t + d \), where \( t \) is the time (in seconds) and \( a, b, c \) and \( d \) are constants. The acceleration of the rotor was obtained by differentiation of the function \( \phi(t) \). The global friction torque obtained varied between 5.62 \( \mu \)Nm and 0.22 \( \mu \)Nm when the rotational speed of the rotor decreasing from 20.5 rad/s to zero, under an axial load of 48 mN. McCarthy et al. [2] experimentally investigated the influence of the speed and of the normal load on the friction torque in a planar-contact encapsulated microball bearing having 0.285 mm diameter steel balls and silicon races. Using the spin-down testing and the hypothesis of the linear dependence between global friction torque and rotational speed, the authors determined the global friction torque for rotational speed between 250 rpm and 5000 rpm and for axial load between 10 mN and 50 mN. Based on the experimental results, authors obtained following empirical power-law model for the global friction torque in the microball bearing [2]:

\[
M = 9 \cdot 10^{-5} \cdot F_N^{0.444} \cdot n
\]

where \( M \) is global friction torque in \( \mu \)Nm, \( F_N \) is the axial load acting on the microball rolling in mN and \( n \) is rotational speed in rot/min. Tan et al. [3] propose a viscoelastic model for friction force developed in a rolling contact between a microball and a plane. This viscoelastic model includes material parameters, ball diameter, normal load and linear speed and was applied for a steel microball having 0.285 mm diameter, loaded with a normal force of 2 mN and rolling on a silicon plane with a linear speed between zero to \( 0.03 \) m/s. The rolling friction torque obtained by this viscoelastic model was between zero to \( 2.2 \cdot 10^{-3} \) \( \mu \)Nm.

Using the integration of the free oscillations equations of a steel microball on a spherical glass surface, Olaru et al. [4] evaluated the rolling friction torque on the basis of the number and amplitude of the experimentally determined microball oscillations. For a steel microball having a diameter of 1 mm, Olaru et al. [4] obtained in dry conditions values for rolling friction torque of \( 0.7 \cdot 10^{-3} \) \( \mu \)Nm at a normal load on microball of 0.04 mN. The experimental results obtained by [1] and [2] refer to the global rolling friction torque in a rotary microball bearing. It is important to be evidenced that in a rotary microball bearing the global friction torque is a result of the rolling friction and of the sliding friction caused both by the pivoting motion of the microballs and the direct contact of the microballs.
To determine only the rolling friction torque in the micro rolling systems the authors developed an analytical model based on the dissipation of the inertial energy of a rotating microdisc in three rolling microballs. Using an original microtribometer with two steel rotating discs and three steel microballs was determined the rolling friction torque in dry conditions for contacts loaded with normal forces of 8.68 mN to 33.2 mN with rotational speed between 30 to 210 rpm.

**ANALYTICAL MODEL**

Figure 1 presents the new micro tribometer. The driving disc 1 is rotated with a constant rotational speed and has a radial groove race. Three microballs are in contact with the race of the disc 1 at the equidistance position (120 degrees). All the three microballs sustain an inertial disc 2 and are normal loaded with a force $Q = \frac{G}{3}$, where $G$ is the weight of the disc 2. When the disc 1 start to rotate with a constant angular speed $\omega_1$, the balls start to roll on the raceway of the disc 1 and start to rotate the inertial disc 2, as a result of rolling friction forces between the balls and the disc 2. As a result of inertial effect the disc 2 is accelerated from zero to the synchronism rotational speed (when $\omega_2 = \omega_1$) in a time $t$, after that the rotational speed of the disc 1 is stopped. The disc 2 has a deceleration process from the constant rotational speed $\omega_{2,0}$ to his completely stop as a result of the friction in the rolling of the three microballs over the two discs.

![Figure 1: General view of the microtribometer](image)

In the deceleration process of the disc 2 when $\omega_2$ decreases from a constant value to zero, following differential equation can be used:

$$J \cdot \frac{d\omega_2}{dt} - 3 \cdot F_2 \cdot r - M_f = 0 \quad (1)$$

were $J$ is the moment of inertia for the disc 2, $F_2$ is the tangential force developed in the contact between a microball and disc 2, $r$ is the radius and $M_f$ is the friction torque developed between the rotating disc 2 and air.

For a disc with inner radius $R_i$, outer radius $R_e$ and a mass $m_d$, the moment of inertia $J$ is determined by relation:

$$J = 0.5 \cdot m_d \cdot (R_i^2 + R_e^2) \quad (2)$$

For a disc having a rotational speed $\omega_2$ in a fluid with a kinematics viscosity $\nu_f$ and a density $\rho_f$, the friction torque $M_f$ can be determined by relation [5]:

$$M_f = 0.5 \cdot K_M \cdot \rho_f \cdot R^5 \cdot \omega^2 \quad (3)$$

where $K_M$ is a coefficient depending on the Reynolds parameter. When the rotational speed of the disc 2 have maximum values between 30 and 210 rpm and the radius $R$ is 0.012 m, the Reynolds parameter have values between 30 and 200 (the kinematics viscosity of the air was considered $\nu_f = 15 \cdot 10^{-6}$ m$^2$/s and the density of the air was considered $\rho_f = 1.18$ kg/m$^3$). For these values of the Reynolds parameter it can be approximated the $K_M$ coefficient by a constant value of 0.5 and equation (3) can be approximated by following equation:

$$M_f = c_f \cdot \omega^2 \quad (4)$$
where the coefficients $c_f$ have an approximate value of $7.3 \times 10^{-11} \text{N} \cdot \text{m} \cdot \text{s}^2$.

As is presented in the figure 2, in the deceleration of the disc 2, on a microball acts following forces in the rotational plane: the tangential contact forces $F_1$ and $F_2$ and the inertial force $F_{ib}$. Also, in the two contacts we consider two rolling friction torques $M_{r1}$ and $M_{r2}$.

The tangential force $F_2$ was determined using the forces and moments equilibrium equations for a microball and results:

$$F_2 = \frac{(M_{r1} + M_{r2})}{d} - \frac{F_{ib}}{2}$$  \hspace{1cm} (5)

where $d$ is the microball diameter.

The inertial force acting in the center of the microball is determined by relation:

$$F_{ib} = m_b \cdot \frac{d\omega_b}{dt} \cdot r$$  \hspace{1cm} (6)

where $m_b$ is the mass of the microball and $\omega_b$ is the angular speed of the microball in the revolution motion around the center of the two discs. Considering the pure rolling motion of the microballs, the angular speed $\omega_b$ can be expressed as $\omega_b = 0.5 \cdot \omega_2$ and the equation (6) can be written:

$$F_{ib} = \frac{m_b \cdot r}{2} \cdot \frac{d\omega_2}{dt}$$  \hspace{1cm} (7)

According to the equations (1), (4), (5) and (7) it can be obtained following differential equation in the deceleration process of the disc 2:

$$\frac{d\omega_2}{dt} = a \cdot (M_{r1} + M_{r2}) + b \cdot \omega_2^2$$  \hspace{1cm} (8)

where $a$ and $b$ are constants defined by relations: $a = \frac{3 \cdot r}{d \cdot (J + \frac{3}{4} \cdot r^2 \cdot m_b)}$, $b = \frac{c_f}{(J + \frac{3}{4} \cdot r^2 \cdot m_b)}$.

To integrate the differential equation (8) two hypothesis was made:

i) In the first hypothesis was considered that the rolling friction torques $M_{r1}$ and $M_{r2}$ are not depending on the rotational speed;

ii) In the second hypothesis was considered that the rolling friction torques $M_{r1}$ and $M_{r2}$ have a linear dependence on rotational speed.

i) Considering that the rolling friction torques $M_{r1}$ and $M_{r2}$ are constants, equation (8) leads to the following solution for $\omega_2$ as function of time:

$$\omega_2(t) = \frac{c}{b} \cdot \text{tg} \left[ -c \cdot t + \text{arctg} \left( \frac{b}{c} \cdot \omega_{2,0} \right) \right]$$  \hspace{1cm} (9)
where \( c = \sqrt{a \cdot b \cdot (M_{r1} + M_{r2})} \) and \( \omega_{2,0} \) is angular rotational speed of the disc 2 at the moment of the stopped the rotation of the disc 1.

Considering that \( \omega_2(t) = \frac{d \phi_2(t)}{dt} \), where \( \phi_2(t) \) is the variation of the angular position of the disc 2 in deceleration process, equation (9) can be integrated and following solution for \( \phi_2(t) \) results:

\[
\phi_2(t) = \frac{-\ln \left[ 1 + \tan \left( -c \cdot t + \arctan \left( \frac{b}{c} \cdot \omega_{2,0} \right) \right) \right]}{2b} + \frac{\ln \left[ 1 + \left( \frac{b}{c} \cdot \omega_{2,0} \right)^2 \right]}{2b}
\]

(10)

ii) Considering that the rolling friction torques \( M_{r1} \) and \( M_{r2} \) have a linear dependence on rotational speed it can be written that \((M_{r1} + M_{r2}) = k \cdot \omega_2\) and differential equation (8) becomes:

\[
\frac{d\omega_2}{dt} = a \cdot k \cdot \omega_2 + b \cdot \omega_2^2
\]

(11)

Equation (11) leads to the following solutions:

\[
\omega_2(t) = \frac{a \cdot k \cdot \exp(-a \cdot k \cdot t + k1)}{1 - b \cdot \exp(-a \cdot k \cdot t + k1)}
\]

(12)

\[
\phi_2(t) = \frac{1}{b} \ln(1 - b \cdot \exp(-a \cdot k \cdot t + k1)) - \frac{1}{b} \ln(1 - b \cdot \exp(k1))
\]

(13)

where \( k1 = \ln \left( \frac{\omega_{2,0}}{a \cdot k + b \cdot \omega_{2,0}} \right) \).

For given dimensions of the microballs and of the two discs, by monitoring the angular position and angular speed of the disc 2 in the deceleration process it is possible to determine the sum of the rolling friction torques \((M_{r1} + M_{r2})\).

Also, having determined the sum of the friction torque it can be determine the tangential force \( F_2 \) by equation (5) and the friction coefficient in the rolling contact \( \mu_r \) with equation:

\[
\mu_r = \frac{F_2}{Q}
\]

(14)

**EXPERIMENTAL INVESTIGATION**

Using the new microtribometer presented in figure 1 a lot of experimental investigations was realized. The microtribometer was mounted on the rotational table of the CETR-UMT Tribometer as in figure 3.

![Figure 3: General view of the experimental equipments](image-url)
To determine the angular acceleration of the disc 2 a high-speed camera Philips SPC900NC/00 VGA CCD with 90 frames/seconds was used to capture the angular position of the disc 2 from the rotational speed $\omega_{2,0}$ to his completely stop. Also, the angular positions of the disc 1 are captured by camera. In figure 4 are presented the registered positions of the disc 2, and of the disc 1, at a short time $t$ after the stop of the disc 1.

The images captured by the camera was processed frame by frame in a PC using Virtual Dub soft and was transferred in AutoCAD to be measured the angular positions $\phi_2$ corresponding to every frame. The camera was installed vertically 150 mm above the disc 2, to minimize the measurement errors. A white mark was placed both on disc 2 and on disc 1 as it can be observed in the figure 4 and the angular positions $\phi_2(t)$ was measured according to the reference position of the mark on the disc 1(position at $t = 0$). The discs 1 and 2 are the steel rings of an axial ball bearing (series 51100) having a rolling path at a radius $r = 8.4$mm and a transversal curvature radius of 2.63 mm. The inertial disc 2 was machined on external surface by electro erosion to reduce the weight to a minimum of $G = 26.05$ mN, and has following dimensions $R_1 = 5$ mm, $R_c = 12$ mm. That means a minimum normal load on every microball $Q = 8.68$ mN. To increase the normal load on the microball a lot of new discs similar to the disc 2 was attached on the disc 2 obtaining following values for the normal load: 8.68 mN, 15 mN, 22.3 mN, 27 mN, 33.2 mN. Three stainless steel microballs having the diameter of 1.588 mm (1/16 inch) was used in the experiments. The roughness of the active surfaces of the two discs and of the balls was measured with Form Talysurf Intra System. Following values of $Ra$ was obtained: rolling path of the disc 1 and 2, $Ra = 0.030$ $\mu$m and ball surface, $Ra = 0.02$ $\mu$m. The tests were realized for the following rotational speed of the disc 2: 30 rpm, 60 rpm, 90 rpm, 120 rpm, 150 rpm, 180 rpm, 210 rpm.

All measurements are performed in steady room environment at a temperature of (18-20)$^\circ$ C and a relative humidity of (40 – 50)%RH. All the tests were realized in dry conditions (without lubricant or condensed water on contact surfaces).

![Figure 4: Determination of the angular position $\phi_2(t)$ of the disc 2](image)

VALIDATION OF THE ANALYTICAL MODELS

Two experimental data were obtained for every experiment: the variation of the angular position $\phi_2(t)$ from the moment of beginning the deceleration process of the disc 2 to his completely stop and the time of the deceleration process. In figure 5 and 6 is presented a typically variation of the angular position $\phi_2(t)$ for a rotational speed of 120 rpm and a normal load $Q = 8.68$ mN experimentally determined. For all the experiment the variations of the angular position of the disc 2, $\phi_2(t)$ are similarly but other time of deceleration and other maximum values were obtained, depending of the initial angular speed $\omega_{2,0}$ and the normal load $Q$ acting on the microballs. Both the two hypothesis was used to validate the experimental results.

i) The hypothesis of constant torque friction was applied for all experiments and we consider that was obtained the good validation with experiments. Using equation (10) was determined the value for the sum of friction torques $(M_{r1} + M_{r2})$ imposing the condition that at the stop of the disc 2 the angular position of this disc to cumulate the experimental determined value. With the sum $(M_{r1} + M_{r2})$ above determined was verified by equation (9) if the angular speed of the disc 2 was stopped after the time experimental determined.

So, in figure 5-a is presented the numerical variation of the angular position of the disc 2 given by equation (10) for a rotational speed of the disc 2 of 120 rpm and a normal load $Q = 8.68$ mN.
The maximum differences between the numerical values obtained by equation (10) and the experimental values are not exceed 5%. In figure 5-b it can be observed the numerical variation of the angular speed $\omega_2(t)$ obtained by equation (9) with a quasi-linear variation from $\omega_{2,0}=12.4$ rad/s to $\omega_{2,0}=0$, in a time $t = 41$ seconds. This deceleration time corresponds to the experimental determined value.

The hypothesis of the linear variation of the friction torque was applied for all experiments. Using equation (13) was determined the value for the sum of friction torques $(M_{r1} + M_{r2})$ imposing the condition that at the stop of the disc 2 the angular position of this disc to cumulate the experimental determined value. With the sum $(M_{r1} + M_{r2})$ above determined was verified by equation (12) the variation of the angular speed of the disc 2.

By comparing the two analytical variations of the angular position $\phi_2(t)$ given by equations (10) and (13) as are presented in figure 7, it can be observed that the equation (10) leads to a variation of angular position with a maximum around of the time $t = 41$ seconds while the equation (13) leads to a continuum increasing of the angular position $\phi_2(t)$. That means theoretically a continuum of rotation over the time of the experimentally stopped of the disc 2.
Our conclusion is that the hypothesis of the constant friction torque can be accepted and leads to a good theoretical model in the interval of the rotational speed between 30 rpm to 210 rpm.

EXPERIMENTAL RESULTS

The sum of the friction torques for all experiments was determined in the hypothesis of the constant friction torque using equations (9) and (10). Considering that the geometry of the contact between microball and the two discs is the same and neglecting the influence of the microball weight (the mass of a microball leads to an additional force $Q_b = 0.165\text{mN}$ in the contact between microball and the disc 1) we can consider that the friction torque between a microball and the disc 1 or 2 is given by relation $M_r = 0.5(M_{r1} + M_{r2})$. In the figure 8 are presented the rolling friction torques $M_r$ for all rotational speeds and normal loads used in the experiments.

It can be observed that between 30 rpm and 150 rpm the friction torque $M_r$ is depending only of the normal load and is not depending of the speed. By increasing the speed from 150 rpm to 210 rpm it can be observe that the friction torque increases with rotational speed, especially by increasing the normal load. The increasing of the friction torque with rotational speed over 150 rpm can be explained by increasing of the rotating disc’s vibration with a supplementary loss of energy. Is important to note that increasing of the normal load is realised by adding supplementary discs on the initial disc 2. Geometrical imperfections of the supplementary discs lead to increasing the vibration level of the rotating disc, and experimentally was observed vibration of the rotating disc.

The friction coefficient determined by equation (14) has values between 0.0002 and 0.0004 that means a dominance of the rolling friction between the microballs and the two discs.

CONCLUSIONS

Was elaborated two analytical models to determine the rolling friction in an original microtribometer. The two models are based on the integration of the differential equation of a rotating disc sustained only by three
microballs. Two hypothesis was considered: i) the friction torque is not depending on the rotational speed in dry contacts and ii) the friction torque has a linear variation with rotational speed.

To validate the hypothesis was realised a lot of experiments with a variation of rotational speed between 30 rpm to 210 rpm and with a variation of a normal load in the rolling contact between 8.68 mN to 33.2 mN. The hypothesis based on the constant friction coefficient was validated as a good hypothesis in dry conditions. The friction torques for all experiments was determined with the analytical model based of the constant friction torque. The numerical values are between 1.8 μN.mm to 7.2 μN.mm.

The rolling friction coefficient realised in all the experiment was between 0.0002 and 0.0004.

ACKNOWLEDGEMENTS

This paper was realised with the support of Grant CNCSIS ID_607 No. 381/1.10.2007 and BRAIN “Doctoral scholarships as an investment in intelligence” project, financed by the European Social Found and Romanian Government.

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